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Reliability-Based Optimization of Active Nonstationary Random Vibration Control

W. Gao*
University of New South Wales, Sydney, New South Wales 2052, Australia

The reliability-based optimization of active bars' placement and feedback gains of a closed-loop control system for stochastic intelligent truss structural active nonstationary random vibration control is presented. The optimal mathematical model with the reliability constraints on the mean square value of structural dynamic displacement and stress response are built based on the maximization of dissipation energy due to control action, where not only the structural physics parameters, geometric dimensions, and damping are considered as random variables, but also the applied forces are considered as nonstationary random excitation. The numerical characteristics of the nonstationary random responses of stochastic intelligent structure are developed by the random factor method. The rationality and validity of the presented model are demonstrated via an engineering example, and the effect of randomness of the structural parameters on the control performance is also examined.

Nomenclature

A	=	bar's cross-sectional area
$A_m^{(e)}$	=	eth passive bar's mass cross-sectional area
$A_P^{(e)}$	=	eth active bars' cross-sectional area
$[B]$	=	element's strain matrix
$[B_1]$	=	input matrix
$C_{E\rho}$	=	correlation coefficient of variables elastic modulus and mass density
$[C]$	=	damping matrix
diag$(\cdot)$	=	diagonal matrix with diagonal elements given in brackets
E	=	elastic modulus
$E^{(e)}$	=	elastic modulus of the eth element
$E_m^{(e)}$	=	eth passive bar's elastic modulus
$E_{33}^{(e)}$	=	eth active bar's elastic modulus
$e_{33}^{(e)}$	=	eth active bar's piezoelectric force/electrical constant
$\{\boldsymbol{F}(\boldsymbol{t})\}$	=	stationary random excitation
$\{\boldsymbol{F}_C(\boldsymbol{t})\}$	=	control force vector
$[G]$	=	gain matrix
$g(t)$	=	time modulation function
$[H(\omega)]$	=	frequency response function matrix
$[H^*(\omega)]$	=	conjugate matrix of $[H(\omega)]$
$[h(t)]$	=	impulse response function matrix
$[K]$, $[M]$	=	global stiffness and mass matrices, respectively
$[K^{(e)}]$, $[M^{(e)}]$	=	eth element's stiffness and mass matrices, respectively
L	=	bar's length
$l_m^{(e)}$	=	eth passive bar's length
$l_P^{(e)}$	=	eth active bars' length
m	=	number of the structural elements
n	=	number of the natural frequencies
$\{\boldsymbol{P}(\boldsymbol{t})\}$	=	stationary random excitation vector
$[R_P(t_1, t_2)]$	=	correlation function matrix of the $\{\boldsymbol{P}(\boldsymbol{t})\}$
$[R_u(t_1, t_2)]$	=	correlation function matrix of structural displacement response
$[R_\sigma^{(e)}(t_1, t_2)]$	=	correlation function matrix of the eth element's stress response
$[S_P(\omega)]$	=	equivalent one-side power spectral density matrix of $\{\boldsymbol{P}(\boldsymbol{t})\}$
$[S_u(\omega, t)]$	=	power spectral density matrix of displacement response
$[S_\sigma^{(e)}(\omega, t)]$	=	power spectral density matrix of the stress response of the eth element
$\{\boldsymbol{u}(\boldsymbol{t})\}$, $\{\dot{\boldsymbol{u}}(\boldsymbol{t})\}$, $\{\ddot{\boldsymbol{u}}(\boldsymbol{t})\}$	=	displacement, velocity, and acceleration vectors, respectively
$\{\boldsymbol{u}(\boldsymbol{t})^{(e)}\}$	=	displacement response of the nodal point of the eth element
$v(t)$	=	time modulation function
$\boldsymbol{Y}(\boldsymbol{t})$	=	output vector
$\varepsilon_{33}^{(e)}$	=	eth active bar's dielectric constant
μ	=	mean value of random variable
ν	=	variation coefficient of random variable
ξ_j	=	jth order mode damping of structure
ρ	=	mass density
$\rho_m^{(e)}$	=	eth passive bar's mass density
$\rho_P^{(e)}$	=	eth active bars' mass density
σ	=	mean variance of random variable
$\{\sigma(t)^{(e)}\}$	=	stress response of the eth element
$[\phi]$	=	normal modal matrix
ψ_σ^2	=	mean square value of structural dynamic stress response
$[\psi_u^2]$	=	mean square value matrix of structural displacement response
ψ_{uk}^2	=	mean square value of the kth degree of freedom of dynamic displacement response
$[\psi_{\sigma^{(e)}}^2]$	=	mean square value matrix of the eth element stress response
ω	=	natural frequency
ω_j	=	jth-order natural frequency

I. Introduction

INTELLIGENT or smart structures are systems whose characteristics can be self-modified during operation to improve their resistance against external disturbances or are to a wide variety of active material and passive structural systems. Piezoelectric structures are one type of smart structure, in which piezoelectric sensors and actuators are used to suppress the structural vibration or shape deformation. A piezoelectric intelligent truss structure is a self-adaptive structure and is used in spacecraft deployable antennas, large antennas, and other important large-scale truss structures. Optimal placement of an piezoelectric active bar is an important segment in the process of intelligent structural vibration control. The locations

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*Postdoctoral Research Fellow, School of Mechanical and Manufacturing Engineering; w.gao@unsw.edu.au.

of active bars in intelligent truss structures affect the validity of active vibration control directly. For example, Rao et al.[1] investigated the discrete optimal actuator location selection problem. They considered this problem in active controlled structures as cast in the framework of a zero-one optimization problem, and a genetic algorithmic approach was developed to solve this zero–one optimization problem. Xu et al.[2] present an optimal design method for placement and gains of actuators and sensors in output feedback control systems. In Ref. 2, a quadratic performance function was minimized using nonlinear programming. Their contribution is the derivation of analytical expressions for the gradients of the performance function. Peng et al.[3] studied the active position control and vibration control of composite beams with distributed piezoelectric sensors and actuators with a finite element modal based on third-order laminate theory. Wang and Wang[4] proposed a controllability index to quantify the controllability factor based on the state-coupled equation of beam structures with piezoelectric actuators, and the index was utilized as an objective function to determine the optimal locations of piezoelectric actuators for vibration control of beam structures. Ray[5] proposed a simple method for optimal control of vibrations of simply supported thin laminated shells integrated with piezoelectric layers. To formulate the optimal control problem, the algorithm for a linear quadratic regulator with output feedback was employed. So far, however, most modeling on optimal placement of active bars in intelligent structures basically belongs to determinate models, that is, all structural parameters, applied loads, and control forces are regarded as determinate ones. Apparently, this kind of model can not reflect the influence of the randomness of intelligent structural parameters, loads, and control forces on the optimal placement of active bars in intelligent structures. In recent years, a great deal of research results of random structures have been published.[6–8] As a matter of fact, there are a lot of engineering structures that have uncertainty in their parameters arising from manufacturing tolerances, materials defects, and variation in operating conditions.

The results of the dynamic response analysis are the important base of the determination of the active bars' location. Because the random dynamic response analysis of stochastic structure is very complicated and difficult, it is only in recent years that the stochastic finite element method based on perturbation technique has been used for solving the dynamic response of structure with random parameters under stationary random excitation. Wall and Bucher[9] researched the dynamic effects of uncertainty in structural properties when the excitation is random by use of perturbation stochastic finite element method (PSFEM). Liu et al.[10] discuss the secular terms resulting from PSFEM in transient analysis of such a random dynamic system. Jensen and Iwan[11] studied the response of systems with uncertain parameters to random excitation by extending the orthogonal expansion method. Zhao and Chen[12] studied the vibration for structures with stochastic parameters to random excitation by using the dynamic Neumann stochastic finite element method, in which the random equation of motion for the structure is transformed into a quasi-static equilibrium equation for the solution of displacement in the time domain. Li and Liao[13] expanded the orthogonal expansion method with the pseudoexcitation method for analyzing the dynamic response of structures with uncertain parameters under external random excitation. Gao and Chen[6] and Gao et al.[8,14] proposed the random factor method (RFM) to solve the problem of the randomness of structural parameters and how to affect the randomness of structural dynamic responses. However, the nonstationary random response of a closed-loop control system for stochastic intelligent structures has not been investigated.

In this paper, stochastic intelligent truss structures are taken as objects of research. The problems of the optimization of an active bar's placement and closed-loop control system's gains are studied, where the applied forces are taken as nonstationary random excitation. Based on the maximization of dissipation energy due to control action, the performance function is developed. Then, the optimal mathematical model with the reliability constraints on the mean square value of structural dynamic displacement and stress response is built. The numerical characteristics of the nonstationary random dynamic responses of stochastic intelligent structures are developed by the random factor method. Through an engineering example, the research on the optimal placement of active bar and the optimization of gains is developed.

II. Mathematical Model of the Optimization Problem

Following the finite element formulation, the equation of motion for an intelligent structure is given by

$$[M]\{\ddot{\boldsymbol{u}}(\boldsymbol{t})\}+[C]\{\dot{\boldsymbol{u}}(\boldsymbol{t})\}+[K]\{\boldsymbol{u}(\boldsymbol{t})\} = v(t)\{\boldsymbol{F}(\boldsymbol{t})\}+[B_1]\{\boldsymbol{F}_C(\boldsymbol{t})\} \quad (1)$$

In the following, Wilson's damping hypothesis (see Ref. 15) will be adopted. With the modal expansion $\{\boldsymbol{u}(\boldsymbol{t})\} = [\phi]\,\{z(t)\}$, the equation of motion takes the form

$$[I]\{\ddot{z}(t)\} + [D]\{\dot{z}(t)\} + [\Omega]\{z(t)\} = [\phi]^T v(t)\{\boldsymbol{F}(\boldsymbol{t})\} + [\phi]^T[B_1]\{\boldsymbol{F}_C(\boldsymbol{t})\} \quad (2)$$

where $[D] = \mathrm{diag}[2\xi_j\omega_j]$, $[\Omega] = \mathrm{diag}[\omega_j^2]$, $j = 1, 2, \ldots, n$, and $[\phi] = [\phi_1 \ldots \phi_n]$.

For active control of the truss bar, a velocity feedback control law is considered. Because each active bar can be considered as a collocated actuator/sensor pair, the output matrix is the transpose of the input matrix. The output vector and control force vector can be expressed as

$$\boldsymbol{Y}(\boldsymbol{t}) = [B_1]^T[\phi]\{\dot{z}(t)\} \quad (3)$$

$$\{\boldsymbol{F}_C(\boldsymbol{t})\} = -[G]\boldsymbol{Y}(\boldsymbol{t}) = -[G][B_1]^T[\phi]\{\dot{z}(t)\} \quad (4)$$

In the following paragraphs, let $[G] = \mathrm{diag}(g_j)$, where g_j is jth element of the principal diagonal, $j = 1, 2, \ldots, n$. Substituting Eq. (4) into Eq. (2) yields the equation of the closed-loop system:

$$[I]\{\ddot{z}(t)\} + \big([D] + [\phi]^T[B_1][G][B_1]^T[\phi]\big)\{\dot{z}(t)\} + [\Omega]\{z(t)\} = [\phi]^T v(t)\{\boldsymbol{F}(\boldsymbol{t})\} \quad (5)$$

In the state-space representation, the equation of motion for the closed-loop system becomes

$$\{\dot{\boldsymbol{u}}(\boldsymbol{t})\} = [A]\{\boldsymbol{u}(\boldsymbol{t})\} \quad (6)$$

where

$$\{\boldsymbol{u}(\boldsymbol{t})\} = \{z(t) \quad \dot{z}(t)\}^T$$

$$[A] = \begin{bmatrix} 0 & [I] \\ -[\Omega] & -([D] + [\phi]^T[B_1][G][B_1]^T[\phi] \end{bmatrix} \quad (7)$$

Both the optimal location of the active bar and the optimal gain of the closed-loop control system are determined such that the total energy dissipated in the system is maximized. Based on the maximization of dissipation energy due to control action, the minimal energy stored in the structures is utilized as the performance, and it can be expressed as

$$J = -\int_0^\infty \{\dot{z}(t)\}^T[\phi]^T[B_1][G][B_1]^T[\phi]\{\dot{z}(t)\}\,\mathrm{d}t \quad (8)$$

When the solution of Eq. (6) is used, $\{\boldsymbol{u}(\boldsymbol{t})\} = \exp([A]t)\{u(0)\}$, Eq. (8) can also be expressed as

$$J = -\{u(0)\}^T \cdot \int_0^\infty e^{[A]^T t}[Q]e^{[A]t}\,\mathrm{d}t \cdot \{u(0)\} \quad (9)$$

where

$$[Q] = \begin{bmatrix} [\Omega] & 0 \\ 0 & [I] \end{bmatrix}$$

When the method described in Ref. 16 is used, the performance function can be expressed as

$$J = -\mathrm{tr}[\bar{K}] \quad (10)$$

where the matrix $[\bar{K}]$ can be obtained by solving the Lyapunov equation,

$$[A]^T[\bar{K}] + [\bar{K}][A] = [Q] \tag{11}$$

For the intelligent truss structure with random parameters and where the loads are nonstationary random excitations, the optimal mathematical model of active bar with the reliability constraints on the mean square value of the structural dynamic stress and displacement response can be built as follows: Find

$$[B_1], [G]$$

$$\min : J = -\mathrm{tr}[\bar{K}] \tag{12}$$

subject to

$$R^*_{\psi^2_\sigma} - P_r\left\{\psi^{2^*}_\sigma - \psi^2_\sigma \geq \delta\right\} \leq 0 \tag{13}$$

$$R^*_{\psi^2_{uk}} - P_r\left\{\psi^{2^*}_{uk} - \psi^2_{uk} \geq \delta\right\} \leq 0 \qquad (k = 1, 2, \ldots, n) \tag{14}$$

$$[B_1] \subset \left[B^*_1\right], \qquad [G] < [G^*] \tag{15}$$

In this model, $[B_1]$ and $[G]$ are design variables. $R^*_{\psi^2_\sigma}$ and $R^*_{\psi^2_{uk}}$ are the given values of reliability of the mean square values of stress and displacement response, respectively. $P_r\{\cdot\}$ is the reliability obtained from the actual calculation and $\psi^{2^*}_\sigma$ and $\psi^{2^*}_{uk}$ are the given limit values of the mean square value of stress and displacement response, respectively. $[B_1]$, $[G]$, $R^*_{\psi^2_\sigma}$, $R^*_{\psi^2_{uk}}$, $P_r\{\cdot\}$, $\psi^{2^*}_\sigma$, and $\psi^{2^*}_{uk}$ can be random variables or deterministic values. Here, ψ^2_σ and ψ^2_{uk} are random variables, δ is the given allowable deviation to avoid the destruction in the structure, which is produced by the lack of the strength or the stiffness. $[B^*_1]$ are the bounds of $[B_1]$, and $[G^*]$ are the upper bounds on the feedback gains.

In the described optimal model, structural dynamic stress and displacement response constraints are expressed by the probability form, which makes the optimal problem difficult to solve. For this reason, the reliability constraints are transformed to be the normal constraints by means of the second-order moment theory of reliability.[17] Thus, the reliability constraints Eq. (13) and (14) can be expressed as, respectively,

$$\beta^*_{\psi^2_\sigma} - \frac{\mu_{\psi^{2^*}_\sigma} - \mu_{\psi^2_\sigma} - \delta_{\psi^2_\sigma}}{\left(\sigma^2_{\psi^{2^*}_\sigma} + \sigma^2_{\psi^2_\sigma}\right)^{\frac{1}{2}}} \leq 0 \tag{13a}$$

$$\beta^*_{\psi^2_{uk}} - \frac{\mu_{\psi^{2^*}_{uk}} - \mu_{\psi^2_{uk}} - \delta_{\psi^2_{uk}}}{\left(\sigma^2_{\psi^{2^*}_{uk}} + \sigma^2_{\psi^2_{uk}}\right)^{\frac{1}{2}}} \leq 0 \qquad (k = 1, 2, \ldots, n) \tag{14a}$$

where $\beta^*_{\psi^2_\sigma} = \Phi^{-1}(R^*_{\psi^2_\sigma})$ and $\beta^*_{\psi^2_{uk}} = \Phi^{-1}(R^*_{\psi^2_{uk}})$ are the given reliability of the mean square value of the structural dynamic stress response and the structural dynamic displacement response of the kth degree of freedom, respectively. $\Phi^{-1}(\cdot)$ are the inverse functions of the distribution of random variables. Here $\delta_{\psi^2_\sigma}$ and $\delta_{\psi^2_{uk}}$ are the given allowable deviations of the mean square value of stress and displacement response, respectively.

III. Intelligent Structural Nonstationary Random Response

Suppose that there are m elements in the intelligent truss structure under consideration. In the structure, any element can be taken as a passive bar or active bar. A piezoelectric bar is utilized as the active bar. To utilize the united form to express the structural stiffness and mass matrices, a kind of mixed element has been constructed. A Boolean algebra value θ is introduced in the mixed element; when $\theta = 0$, the mixed element is the active element bar and when $\theta = 1$, the mixed element is the passive element bar. In the following paragraphs, expressions of the stiffness matrix $[K]$ and mass matrix $[M]$ of intelligent truss structures in global coordinate will be developed by means of this kind of mixed element:

$$[K] = \sum_{e=1}^{m}\left[K^{(e)}\right] = \sum_{e=1}^{m}\left\{\left[\theta\frac{E^{(e)}_m A^{(e)}_m}{l^{(e)}_m} + (1-\theta)\frac{E^{(e)}_{33} + \left(e^{(e)}_{33}\right)^2 \big/ \varepsilon^{(e)}_{33}}{l^{(e)}_P}A^{(e)}_P\right][\bar{G}]\right\} \tag{16}$$

$$[M] = \sum_{e=1}^{m}\left[M^{(e)}\right] = \sum_{e=1}^{m}\left\{\frac{1}{2}\left(\theta\rho^{(e)}_m A^{(e)}_m l^{(e)}_m + (1-\theta)\rho^{(e)}_P A^{(e)}_P l^{(e)}_P\right)[I]\right\} \tag{17}$$

where $[\bar{G}]$ is a 6×6 matrix. $[I]$ is a six-order identity matrix.

Introduce another expression as follows:

$$E^{(e)}_P = E^{(e)}_{33} + \left(e^{(e)}_{33}\right)^2 \big/ \varepsilon^{(e)}_{33} \tag{18}$$

$E^{(e)}_P$ just is a generalized elastic modulus of piezoelectric active bars taking into consideration the mechanic–electronic coupling effect.

Substituting Eq. (18) into Eq. (16) yields

$$[K] = \sum_{e=1}^{m}\left[K^{(e)}\right] = \sum_{e=1}^{m}\left\{\left[\theta\frac{E^{(e)}_m A^{(e)}_m}{l^{(e)}_m} + (1-\theta)\frac{E^{(e)}_P A^{(e)}_P}{l^{(e)}_P}\right][\bar{G}]\right\} \tag{19}$$

In the closed-loop control system, because the production and response process of $\{F_P(t)\}$ is determined by the nonstationary random excitation, $\{\boldsymbol{F}_C(\boldsymbol{t})\}$ is the nonstationary random force vector, too, and these two variables have full positive correlation.

Let

$$g(t)\{\boldsymbol{P}(\boldsymbol{t})\} = v(t)\{\boldsymbol{F}(\boldsymbol{t})\} + [B_1]\{\boldsymbol{F}_C(\boldsymbol{t})\} \tag{20}$$

Then, Eq. (1) can be expressed as

$$[M]\{\ddot{\boldsymbol{u}}(\boldsymbol{t})\} + [C]\{\dot{\boldsymbol{u}}(\boldsymbol{t})\} + [K]\{\boldsymbol{u}(\boldsymbol{t})\} = g(t)\{\boldsymbol{P}(\boldsymbol{t})\} \tag{21}$$

Equation (21) is a set of differential equations coupled to each other. Its formal solution can be obtained in terms of the decoupling transform and Duhamel integral, that is,

$$\{\boldsymbol{u}(\boldsymbol{t})\} = \int_0^t [\phi][h(t-\tau)][\phi]^T g(\tau)\{P(\tau)\}\,\mathrm{d}\tau \tag{22}$$

where

$$[h(t)] = \mathrm{diag}[h_j(t)] \tag{23}$$

$$h_j(t) = \begin{cases} (1/\bar{\omega}_j)\exp(-\xi_j\omega_j t)\sin\bar{\omega}_j t, & t \geq 0 \\ 0, & t < 0 \end{cases} \qquad (j = 1, 2, \ldots, n) \tag{24}$$

where $\bar{\omega}_j = \omega_j(1 - \xi^2_j)^{1/2}$.

From Eq. (22), the correlation function matrix of the structural displacement response can be obtained:

$$[R_u(t_1, t_2)] = E(\{u(t_1)\}\{u(t_2)\}^T) = \int_0^{t_1}\int_0^{t_2}[\phi][h(t-\tau_1)][\phi]^T g(\tau_1)[R_P(\tau_1, \tau_2)]g(\tau_2)[\phi] \times [h(t-\tau_2)]^T[\phi]^T\,\mathrm{d}\tau_1\,\mathrm{d}\tau_2 \tag{25}$$

When a Fourier transformation to $[R_u(t_1, t_2)]$ is performed, the power spectral density matrix of the structural displacement response can be obtained:

$$[S_u(\omega,t)] = [\phi][H(\omega)][\phi]^T g(t_1)[S_P(\omega)]g(t_2)[\phi][H^*(\omega)][\phi]^T \tag{26}$$

where

$$[H(\omega)] = \mathrm{diag}[H_j(\omega)] \tag{27}$$

$$H_j(\omega) = \left[1\Big/\left(\omega_j^2 - \omega^2 + i\cdot 2\xi_j\omega_j\omega\right)\right] \qquad \left(i=\sqrt{-1}\right) \qquad (j=1,2,\ldots,n) \tag{28}$$

After the integration of $[S_u(\omega,t)]$ within the frequency domain, the mean square value matrix of the structural displacement response can be obtained:

$$\left[\psi_u^2\right] = \int_0^\infty [S_u(\omega,t)]\,\mathrm{d}\omega = \int_0^\infty [\phi][H(\omega)][\phi]^T \times g(t_1)[S_P(\omega)]g(t_2)[\phi][H^*(\omega)][\phi]^T\,\mathrm{d}\omega \tag{29}$$

Then the mean square value of the kth degree of freedom of the structural dynamic displacement response can be expressed as

$$\psi_{uk}^2 = \int_0^\infty \phi_k[H(\omega)][\phi]^T g(t_1)[S_P(\omega)]g(t_2)[\phi][H^*(\omega)]\phi_k^T\,\mathrm{d}\omega \qquad (k=1,2,\ldots,n) \tag{30}$$

where ϕ_k is the kth line vector of the matrix $[\phi]$.

According to the relationship between node displacement and element stress, the stress response of the eth element in the truss structure can be expressed as

$$\left\{\sigma(t)^{(e)}\right\} = E^{(e)}\cdot[B]\cdot\left\{\boldsymbol{u}(\boldsymbol{t})^{(e)}\right\} \qquad (e=1,2,\ldots,m) \tag{31}$$

From Eq. (31), the correlation function matrix of the eth element stress response can be obtained,

$$\left[R_\sigma^{(e)}(t_1,t_2)\right] = \tilde{E}\left(\left\{\sigma(t)^{(e)}\right\}\left\{\sigma(t+\tau)^{(e)}\right\}^T\right) = \tilde{E}^{(e)}[B]\left[R_u^{(e)}(t_1,t_2)\right][B]^T\tilde{E}^{(e)} \tag{32}$$

Furthermore, the power spectral density matrix of the stress response of the eth element can be obtained:

$$\left[S_\sigma^{(e)}(\omega,t)\right] = E^{(e)}[B]\left[S_u^{(e)}(\omega,t)\right][B]^T E^{(e)} \tag{33}$$

Then, the mean square value matrix of the eth element stress response can be expressed as

$$\left[\psi_{\sigma^{(e)}}^2\right] = E^{(e)}[B]\left[\psi_{u^{(e)}}^2\right][B]^T E^{(e)} \tag{34}$$

IV. Numerical Characteristics of Intelligent Structural Nonstationary Random Response

A. Numerical Characteristics Analysis of Dynamic Characteristics by RFM[14]

The randomness of ξ_j, $\rho_m^{(e)}$, $\rho_P^{(e)}$, $A_m^{(e)}$, $A_P^{(e)}$, $l_m^{(e)}$, $l_P^{(e)}$, $E_m^{(e)}$, and $c_{33}^{(e)}$ are considered simultaneously. From Eq. (18), it can be obtained easily that $E_P^{(e)}$ is random variable. The randomness of physical parameters and geometric dimensions will lead the structural matrices $[K]$ and $[M]$ having randomness. Moreover, the uncertainty of the structural matrices $[K]$ and $[M]$ will lead to the structural natural frequency ω_j and natural modal shape $\{\phi\}$ having uncertainty.

In the following paragraph, the computing expression of the mean value and mean variance (standard deviation) of jth-order natural frequency can be deduced by means of the algebra synthesis method:

$$\mu_{\omega_j} = \omega_j^{\#}\left\{\left[1+\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2 - c_{E\rho}\cdot\nu_E\cdot\left(\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2\right)^{\frac{1}{2}}\right]^2 - \tfrac{1}{2}\left[\nu_E^2+\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2 - 2c_{E\rho}\cdot\nu_E\cdot\left(\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2\right)^{\frac{1}{2}}\right]\right\}^{\frac{1}{4}} \tag{35}$$

$$\sigma_{\omega_j} = \omega_j^{\#}\Bigg\{\left[1+\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2 - c_{E\rho}\cdot\nu_E\cdot\left(\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2\right)^{\frac{1}{2}}\right] - \left\{\left[1+\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2 - c_{E\rho}\cdot\nu_E\cdot\left(\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2\right)^{\frac{1}{2}}\right]^2 - \tfrac{1}{2}\left[\nu_E^2+\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2 - 2c_{E\rho}\cdot\nu_E \cdot\left(\nu_Z^2+\nu_\rho^2+\nu_Z^2\nu_\rho^2\right)^{\frac{1}{2}}\right]\right\}^{\frac{1}{2}}\Bigg\}^{\frac{1}{2}}$$

$$\nu_z = \sqrt{4\nu_L^2+2\nu_L^4}\Big/\left(1+\nu_L^2\right) \tag{36}$$

where $\omega_j^{\#}$ can be obtained by the structural conventional dynamic characteristic analysis for deterministic structures.

Likewise, the randomness of each element ϕ_{ij} of the modal matrix is equal and can be expressed as

$$\mu_{\phi_{ij}} = \phi_{ij}^{\#}\cdot\left[1+\tfrac{1}{2}\left(\nu_\rho^2+\nu_A^2+\nu_l^2+\nu_\rho^2\nu_A^2+\nu_\rho^2\nu_l^2+\nu_A^2\nu_l^2+\nu_\rho^2\nu_A^2\nu_l^2\right)\right]^{\frac{1}{4}} \tag{37}$$

$$\sigma_{\phi_{ij}} = \phi_{ij}^{\#}\cdot\left\{1+\nu_\rho^2+\nu_A^2+\nu_l^2+\nu_\rho^2\nu_A^2+\nu_\rho^2\nu_l^2+\nu_A^2\nu_l^2+\nu_\rho^2\nu_A^2\nu_l^2 - \left[1+\tfrac{1}{2}\left(\nu_\rho^2+\nu_A^2+\nu_l^2+\nu_\rho^2\nu_A^2+\nu_\rho^2\nu_l^2+\nu_A^2\nu_l^2+\nu_\rho^2\nu_A^2\nu_l^2\right)\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}} \tag{38}$$

$$\nu_{\phi_{ij}} = \sigma_{\phi_{ij}}\Big/\mu_{\phi_{ij}} \tag{39}$$

The deterministic values (mean values) of the natural modal shape and the modal matrix can be obtained by means of the conventional dynamic analysis method.

B. Numerical Characteristics of the Nonstationary Random Response of Stochastic Intelligent Structures

The randomness of the structural damping, dynamic characteristics, and the nonstationary random excitation will lead the structural dynamic response (dynamic displacement and dynamic stress) of the closed-loop control system having randomness. In the following paragraphs, expressions of the numerical characteristics of the structural dynamic response random variables will be derived.

From Eq. (30), the mean value and mean variance of the mean square value of the kth degree of freedom of the structural dynamic displacement response can be deduced by means of the random variable's functional moment method,

$$\mu_{\psi_{uk}^2} = \int_0^\infty \mu_{\vec{\phi}_k}\mu_{[H(\omega)]}\mu_{[\phi]^T}g(t_1)\mu_{[S_P(\omega)]}g(t_2)\mu_{[\phi]}\mu_{[H^*(\omega)]}\mu_{\vec{\phi}_k^T}\,\mathrm{d}\omega \tag{40}$$

$$\begin{aligned}\sigma_{\psi_{uk}^2} = \Bigg\{\int_0^\infty \Big\{&\left[\sigma_{\vec{\phi}_k}\mu_{[H(\omega)]}\mu_{[\phi]^T}g(t_1)\mu_{[S_P(\omega)]}g(t_2)\mu_{[\phi]}\mu_{[H^*(\omega)]}\mu_{\vec{\phi}_k^T}\right]^2\\ &+\left[\mu_{\vec{\phi}_k}\sigma_{[H(\omega)]}\mu_{[\phi]^T}g(t_1)\mu_{[S_P(\omega)]}g(t_2)\mu_{[\phi]}\mu_{[H^*(\omega)]}\mu_{\vec{\phi}_k^T}\right]^2\\ &+\left[\mu_{\vec{\phi}_k}\mu_{[H(\omega)]}\sigma_{[\phi]^T}g(t_1)\mu_{[S_P(\omega)]}g(t_2)\mu_{[\phi]}\mu_{[H^*(\omega)]}\mu_{\vec{\phi}_k^T}\right]^2\\ &+\left[\mu_{\vec{\phi}_k}\mu_{[H(\omega)]}\mu_{[\phi]^T}g(t_1)\mu_{[S_P(\omega)]}g(t_2)\sigma_{[\phi]}\mu_{[H^*(\omega)]}\mu_{\vec{\phi}_k^T}\right]^2\\ &+\left[\mu_{\vec{\phi}_k}\mu_{[H(\omega)]}\mu_{[\phi]^T}g(t_1)\mu_{[S_P(\omega)]}g(t_2)\mu_{[\phi]}\sigma_{[H^*(\omega)]}\mu_{\vec{\phi}_k^T}\right]^2\\ &+\left[\mu_{\vec{\phi}_k}\mu_{[H(\omega)]}\mu_{[\phi]^T}g(t_1)\mu_{[S_P(\omega)]}g(t_2)\mu_{[\phi]}\mu_{[H^*(\omega)]}\sigma_{\vec{\phi}_k^T}\right]^2\Big\}\,\mathrm{d}\omega\Bigg\}^{\frac{1}{2}}\end{aligned} \qquad (k=1,2,\ldots,n) \tag{41}$$

$$\sigma_{[H(\omega)]} = \operatorname{diag}\left\{ \frac{\left\{\left[\left(2\mu_{\omega_j} + i\cdot 2\mu_{\xi_j}\omega\right)\cdot\sigma_{\omega_j}\right]^2 + \left[\left(i\cdot 2\mu_{\omega_j}\omega\right)\cdot\sigma_{\xi_j}\right]^2\right\}^{\frac{1}{2}}}{\left(\mu_{\omega_j}^2 - \omega^2 + i\cdot 2\mu_{\xi_j}\mu_{\omega_j}\omega\right)^2} \right\} \quad (j = 1, 2, \ldots, n) \tag{42}$$

From Eqs. (40) and (41), the variation coefficient of the mean square value of the kth degree of freedom of the structural dynamic displacement response can be obtained:

$$\nu_{\psi_{uk}^2} = \sigma_{\psi_{uk}^2} \big/ \mu_{\psi_{uk}^2} \tag{43}$$

From Eq. (34), the expressions of numerical characteristics of the element stress response can be deduced by means of the algebra synthesis method:

$$\mu_{[\psi_{\sigma(e)}^2]} = \left(\mu_E^2 + \sigma_E^2\right)\cdot[B]\cdot\mu_{\left[\psi_{u(e)}^2\right]}\cdot[B]^T \qquad (e = 1, 2, \ldots, m) \tag{44}$$

$$\sigma_{\left[\psi_{\sigma(e)}^2\right]} = \left\{\left(\mu_E^2 + \sigma_E^2\right)^2\cdot\left([B]\cdot\sigma_{\left[\psi_{u(e)}^2\right]}\cdot[B]^T\right)^2 + \left(4\mu_E^2\sigma_E^2 + 2\sigma_E^4\right) \cdot\left([B]\cdot\mu_{\left[\psi_{u(e)}^2\right]}\cdot[B]^T\right)^2 + \left(4\mu_E^2\sigma_E^2 + 2\sigma_E^4\right) \cdot\left([B]\cdot\sigma_{\left[\psi_{u(e)}^2\right]}\cdot[B]^T\right)^2\right\}^{\frac{1}{2}} \qquad (e = 1, 2, \ldots, m) \tag{45}$$

From Eqs. (44) and (45), the variation coefficient of the mean square value of the eth element stress response can be obtained:

$$\nu_{\left[\psi_{\sigma(e)}^2\right]} = \sigma_{\left[\psi_{\sigma(e)}^2\right]} \Big/ \mu_{\left[\psi_{\sigma(e)}^2\right]} \qquad (e = 1, 2, \ldots, m) \tag{46}$$

V. Example

A 20-bar planar intelligent truss structure shown in Fig. 1 is used to illustrate the method. The material properties of the active and passive bars are given in Table 1.

To solve the optimal problem, two steps are adopted. In the first step, the reliability constraints of dynamic stress and displacement are neglected and the feedback gains are kept constant. Then, each element bar is taken as an active bar in turn, and the corresponding performance function value is calculated. Based on the computational results for the dissipated energy, the optimal location of the active bar can be determined. In the second step, after the optimal placement of the active bar is obtained, the reliability constraints are imposed, and the optimization of feedback gain, that is, minimization of feedback gain, will be developed.

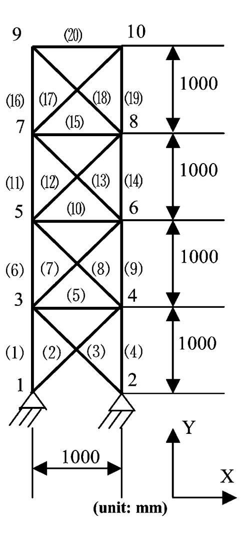

Fig. 1 Planar intelligent truss structure, 20 bars.

Table 1 Intelligent truss structure's physical parameters

Property	Active bar (PZT-4)	Passive bar (steel)
Mean value of mass density ρ, kg/m^3	7600	7800
Mean value of elastic modulus c_{33}, N/m^2	8.807×10^{10}	2.1×10^{11}
Piezoelectric force/electric constant e_{33}, C/m^2	18.62	——
Dielectric constant ε_{33}, C/V · m	5.92×10^{-9}	——
Cross-section area A, m^2	3.0×10^{-4}	3.0×10^{-4}

Table 2 Computational results of performance function, $g = 50$

Element	Value of J	Element	Value of J
1	−123.06	11	−71.17
2	−117.83	12	−60.02
3	−117.83	13	−60.02
4	−123.06	14	−71.17
5	−96.45	15	−58.33
6	−85.76	16	−44.29
7	−78.49	17	−36.75
8	−78.49	18	−36.75
9	−85.76	19	−44.29
10	−68.02	20	−31.21

A. Optimal Placement of Active Bar

For the first step, and letting the closed loop control system feedback gains be $g = g_j = 50$, each element bar is taken as the active bar in turn; the corresponding performance function value is given in Table 2.

From Table 2, it can be seen that, if the first or fourth element is used as the active bar, the active control performance of the intelligent truss structure is the best. The effect of active vibration control of the intelligent truss structure is the worst if the 20th element is used as the active bar.

B. Optimization of Feedback Gains

For comparison, the 1st element and the 20th element are utilized as active bars, respectively. When the reliability constraints are considered, the optimization of feedback gain is developed. The elastic modulus E, mass density ρ, bars' length l, bars' cross-sectional area A, and structural damping ξ_j are all random variables. Let $\mu_{\xi_j} = \mu_\xi = 0.01$. A ground level acceleration act on the structure, $\boldsymbol{F}(\boldsymbol{t})$, is a Gauss stationary random process, and its mean value is zero. Its self-power spectral density can be expressed as[14]

$$S_{FF}(\omega) = \frac{1 + 4(\xi_g\omega/\omega_g)^2}{\left(1 - \omega^2\big/\omega_g^2\right)^2 + 4(\xi_g\omega/\omega_g)^2} S_0 \tag{47}$$

where $\omega_g = 16.5$, $\xi_g = 0.7$, and $S_0 = 13.7$ cm^2/s^3. The time modulation function $v(t)$ can be expressed as

$$v(t) = \begin{cases} (t/t_b)^2, & 0 \le t < t_b \\ 1.0, & t_b \le t < t_c \\ \exp[-\alpha(t - t_c)], & t \ge t_c \end{cases} \tag{48}$$

where $t_b = 7.1$ s, $t_c = 19.5$ s, and $\alpha = 0.16$.

The deterministic model and random model are all adopted in the computational process. In the determinate model, the mean values of all random variables are regarded as determinate quantities, and their variation coefficients are taken as zero. In the random model, to investigate the effect of the dispersal degree of random variables E, ρ, l, A and ξ_j on the optimal results, the values of the variation coefficients of parameters E, ρ, l, A, ξ_j, ψ_σ^{2*}, and ψ_{uk}^{2*} are taken as two groups, respectively. In group 1, $\nu_E = \nu_\rho = \nu_l = \nu_A = \nu_{\xi_j} = \nu_{\psi_\sigma^{2*}} = \nu_{\psi_{uk}^{2*}} = 0.02$. In

Table 3 Computational results of feedback gains

Design variable	1st element utilized as active bar: Original value	Determinate model	Random model 1	Random model 2	16th element utilized as active bar: Original value	Determinate model	Random model 1	Random model 2
G	50	49.27	69.51	85.04	50	62.23	81.79	105.77
G^{a}			69.53[a]	85.07[a]			81.82[a]	105.83[a]
$\mu_{\psi_\sigma^2}$, MPa^2	1737.6	1999.7	1582.8	1253.4	2179.3	1999.1	1582.5	1252.9
$\mu_{\psi_\sigma^2}$, MPa^{2a}			158.31[a]	1253.8[a]			1582.9[a]	1253.5[a]
$\mu_{\psi_{uk}^2}$, mm^2	2.7493	2.8454	2.3741	1.9018	3.3079	2.8468	2.3739	1.9014
$\mu_{\psi_{uk}^2}$, mm^{2a}			2.3743[a]	1.9022[a]			2.3744[a]	1.9019[a]
$R_{\psi_\sigma^2}$		0.47	0.98	0.98		0.47	0.98	0.98
$R_{\psi_{uk}^2}$		0.51	0.95	0.95		0.51	0.95	0.95

[a]Dynamic analysis by Monte Carlo simulation method.

group 2, $\nu_E = \nu_\rho = \nu_l = \nu_A = \nu_{\xi_j} = \nu_{\psi_\sigma^{2*}} = \nu_{\psi_{uk}^{2*}} = 0.2$. Here $\mu_{\psi_\sigma^{2*}}$ and $\mu_{\psi_{uk}^{2*}}$ are all random variables. Their mean values are $\mu_{\psi_\sigma^{2*}} = \pm 2000$ MPa^2 and $\mu_{\psi_{uk}^{2*}} = 3.0000$ mm^2, respectively. In addition, $R^*_{\psi_\sigma^2} = R^*_{\psi_{uk}^2} = 0.95$. The corresponding optimal results are given in Table 3. In addition, to verify our method, the optimal results are given in Table 3, where the stochastic structural nonstationary random responses are obtained by the Monte Carlo simulation method.

From Table 3, it can be seen easily that the optimal results of the method proposed in this paper agree with those of the random structural no-stationary random responses analyzed by Monte Carlo simulation method, by which the validity of our method is verified.

VI. Conclusions

1) The optimal results of deterministic models and random models are different. The optimal results of a deterministic model fulfill the normal constraints, but the results cannot fulfill the reliability constraints. From the probability standpoint, the former is the infeasible solution of the latter one in a common instance.

2) The effectiveness of using the active element is strongly dependent on its location in the truss structure. The randomness of the structural parameters obviously affects the optimal results of feedback gains. The optimal value of the feedback gains will increase remarkably along with the increase of the degree of dispersal of these random variables.

3) The example shows that the areas of the system where the most energy is stored are the optimal locations of an active bar to maximize its damping effect. The example also shows that the model and solving method presented in this paper are rational and feasible.

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M. Ahmadian
Associate Editor